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Local Duality Predictions for $x \sim 1$ Structure Functions

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Abstract

Recent data on the proton F_2 structure function in the resonance region suggest that local quark-hadron duality works remarkably well for each of the low-lying resonances, including the elastic, to rather low values of Q^2. We discuss model-independent relations between structure functions at $x \sim 1$ and elastic electromagnetic form factors. The $x \to 1$ behavior of the nucleon polarization asymmetries and the neutron to proton structure function ratios is studied as a function of Q^2 using available data on the electric and magnetic form factors.

The nucleon's deep-inelastic structure functions and elastic form factors parameterize fundamental information about its quark substructure. Both reflect dynamics of the internal quark wave functions describing the same physical ground state, albeit in different kinematic regions. Recent work on generalized parton distributions [1] has provided a unifying framework within which both form factors and structure functions can be simultaneously embedded.

Exploration of the structure function—form factor interface is actually as old as the first deep-inelastic scattering experiments themselves. In the early 1970s the inclusive-exclusive connection was studied in the context of deep-inelastic scattering in the resonance region and the onset of scaling behavior. In their pioneering paper, Bloom and Gilman [2] observed that the inclusive F_2 structure function at low W generally follows a global scaling curve which describes high W data, to which the resonance structure function averages. Furthermore, the equivalence of the averaged resonance and scaling structure functions appears to hold for each resonance, over restricted regions in W, so that the resonance—scaling duality also exists locally.

More recently, high precision data on the F_2 structure function from Jefferson Lab [3] have confirmed the original observations of Bloom and Gilman, demonstrating that local duality works remarkably well for each of the low-lying resonances, including surprisingly the elastic, to rather low values of Q^2. In this Letter we examine local duality for the elastic case more closely, and derive *model-independent* relations between structure functions at $x \sim 1$ and elastic electromagnetic form factors. Using the most recent data on the electric and magnetic form factors, we apply local duality to make quantitative predictions for the asymptotic behavior of unpolarized and polarized structure function ratios, which can then be compared directly against proton and neutron data at large x.

Following Bloom and Gilman's empirical observations, de Rújula, Georgi and Politzer [4] pointed out that global duality can be understood from an operator product expansion of QCD moments of structure functions. Expanding the F_2 moments in a power series in $1/Q^2$,

$$\int_0^1 d\xi\, \xi^n F_2\left(\xi, Q^2\right) = \sum_{k=0}^{\infty} \left(\frac{n\Lambda^2}{Q^2}\right)^k A_n^{(k)}\left(\alpha_s(Q^2)\right), \tag{1}$$

where Λ is some mass scale, and the Nachtmann scaling variable $\xi = 2x/(1+\sqrt{1+x^2/\tau})$, with $\tau = Q^2/4M^2$, takes into account target mass corrections, one can attribute the existence of global duality to the relative size of higher twists in deep-inelastic scattering. The Q^2 dependence of the coefficients $A_n^{(k)}$ arises only through $\alpha_s(Q^2)$ corrections, and the higher twist matrix elements $A_n^{(k>0)}$ are expected to be of the same order of magnitude as the leading twist term $A_n^{(0)}$. The weak Q^2 dependence of the low F_2 moments can then be interpreted as indicating that higher twist ($1/Q^{2k}$ suppressed) contributions are either small or cancel.

In subsequent analyses, Ji et al. [5,6] showed that matrix elements of higher twist operators, which parameterize information on quark-gluon correlations, can be extracted from moments of structure functions measured in the resonance region at intermediate Q^2, where the $1/Q^2$ corrections are neither negligible nor overwhelming.

Although global Bloom–Gilman duality of low structure function moments can be analyzed systematically within a perturbative operator product expansion, an elementary understanding of local duality's origins is more elusive. This problem is closely related to the question of how to build up a scaling ($\approx Q^2$ independent) structure function from resonance contributions [7], each of which is described by a form factor $G_R(Q^2)$ that falls off as some power of $1/Q^2$.

To illustrate the interplay between resonances and scaling functions, one can observe [2,8] that (in the narrow resonance approximation) if the contribution of a resonance of mass M_R to the F_2 structure function at large Q^2 is given by $F_2^{(R)} = 2M\nu\left(G_R(Q^2)\right)^2\, \delta(W^2 - M_R^2)$, then a form factor behavior $G_R(Q^2) \sim (1/Q^2)^n$ translates into a structure function $F_2^{(R)} \sim (1-x_R)^{2n-1}$, where $x_R = Q^2/(M_R^2 - M^2 + Q^2)$. On purely kinematical grounds, therefore, the resonance peak at x_R does not disappear with increasing Q^2, but rather moves towards $x = 1$.

For elastic scattering, the connection between the $1/Q^2$ power of the elastic form factors at large Q^2 and the $x \to 1$ behavior of structure functions was first established by Drell and

Yan [9] and West [10]. Although it was derived before the advent of QCD, the Drell-Yan-West form factor–structure function relation can be expressed in perturbative QCD language in terms of hard gluon exchange. The pertinent observation is that deep-inelastic scattering at $x \sim 1$ probes a highly asymmetric configuration in the nucleon in which one of the quarks goes far off-shell after exchange of at least two hard gluons in the initial state; elastic scattering, on the other hand, requires at least two gluons in the final state to redistribute the large Q^2 absorbed by the recoiling quark [11]. The duality relations between structure functions at $x \sim 1$ and resonances were further elaborated in the context of perturbative QCD by Carlson and Mukhopadhyay in Ref. [8]. More recently, interest in large-x structure functions has arisen in connection with polarization asymmetry $A_1 = g_1/F_1$, and the F_2^n/F_2^p ratio, whose $x \to 1$ limits reflect mechanisms for the breaking of spin-flavor SU(6) symmetry in the nucleon [12,13].

If the inclusive–exclusive connection via local duality is taken seriously, one can use measured structure functions in the resonance region at large ξ to directly extract elastic form factors [4]. Conversely, empirical electromagnetic form factors at large Q^2 can be used to predict the $x \to 1$ behavior of deep-inelastic structure functions [2]. To quantify this connection, we begin by noting that the elastic contributions to the inclusive spin-averaged structure functions can be expressed through electric and magnetic form factors as [8]:

$$F_1^{el} = M\tau\, G_M^2\, \delta\left(\nu - \frac{Q^2}{2M}\right), \tag{2a}$$

$$F_2^{el} = \frac{2M\tau}{1+\tau}\left(G_E^2 + \tau G_M^2\right)\, \delta\left(\nu - \frac{Q^2}{2M}\right), \tag{2b}$$

while for spin-dependent structure functions [6,8]:

$$g_1^{el} = \frac{M\tau}{1+\tau} G_M\left(G_E + \tau G_M\right)\delta\left(\nu - \frac{Q^2}{2M}\right), \tag{2c}$$

$$g_2^{el} = \frac{M\tau^2}{1+\tau} G_M\left(G_E - G_M\right)\delta\left(\nu - \frac{Q^2}{2M}\right). \tag{2d}$$

Following de Rújula et al. [4], one can integrate Eqs.(2) over ξ between pion threshold and 1, allowing the "localized" moments of structure functions to be expressed in terms of

elastic form factors. Resonance data on the F_2 structure function at large ξ were used by de Rújula et al. [4], and more recently Niculescu et al. [3], to extract the proton's G_M form factor (assuming that the ratio G_E/G_M is sufficiently constrained). They found better than $\sim 30\%$ agreement over a large range of Q^2.

Applying the argument in reverse, one can formally differentiate the lowest ($n = 0$) "localized" moments with respect to Q^2 [2] to express structure functions at $x \sim 1$ in terms of Q^2 derivatives of elastic form factors:

$$F_1 \propto \frac{dG_M^2}{dQ^2}\,, \tag{3a}$$

$$F_2 \propto \frac{G_M^2 - G_E^2}{4M^2(1+\tau)^2} + \frac{1}{1+\tau}\left(\frac{dG_E^2}{dQ^2} + \tau\frac{dG_M^2}{dQ^2}\right) \to \frac{dG_M^2}{dQ^2} \quad \text{as } \tau \to \infty\,, \tag{3b}$$

$$g_1 \propto \frac{G_M\left(G_M - G_E\right)}{4M^2(1+\tau)^2} + \frac{1}{1+\tau}\left(\frac{d(G_E G_M)}{dQ^2} + \tau\frac{dG_M^2}{dQ^2}\right) \to \frac{dG_M^2}{dQ^2} \quad \text{as } \tau \to \infty\,, \tag{3c}$$

$$g_2 \propto \frac{G_M\left(G_M - G_E\right)}{4M^2(1+\tau)^2} + \frac{\tau}{1+\tau}\left(\frac{d(G_E G_M)}{dQ^2} + \frac{dG_M^2}{dQ^2}\right) \to \frac{d}{dQ^2}\left(G_M^2 + G_E G_M\right) \quad \text{as } \tau \to \infty\,, \tag{3d}$$

where the $Q^2 \to \infty$ limits have been explicitly indicated. It is interesting to observe that asymptotically each of the structure functions F_1, F_2 and g_1 are determined by the slope of the square of the magnetic form factor, while g_2 (which in deep-inelastic scattering is associated with higher twists) is determined by a combination of G_E and G_M.

Equations (3) allow the $x \sim 1$ behavior of structure functions to be predicted from empirical electromagnetic form factors. Of particular interest is the $x \to 1$ behavior of the polarization asymmetry, A_1, which at large Q^2 is given by the ratio of spin-dependent to spin-averaged structure functions, $A_1 = g_1/F_1$. From spin-flavor SU(6) symmetry one expects $A_1 = 5/9$ for the proton, and $A_1 = 0$ for the neutron. A number of models which incorporate SU(6) breaking [12,14], through either perturbative or non-perturbative mechanisms, suggest that $A_1 \to 1$ as $x \to 1$, in dramatic contrast to the SU(6) predictions, especially for the neutron.

Using the parameterization of global form factor data from Ref. [15], the proton and neutron asymmetries arising from the local quark–hadron duality relations (3) are shown in Fig. 1 as a function of Q^2. One sees that while at low Q^2 the asymmetries are qualitatively consistent with the SU(6) expectations, the trend as Q^2 increases is for both asymmetries to approach unity, as evident from Eqs.(3). This is consistent with the operator product expansion interpretation of de Rújula et al. [4] in which duality should be a better approximation with increasing Q^2.

Unfortunately the current data on A_1 extend only out to an average $\langle x \rangle \sim 0.5$, and are inconclusive about the $x \to 1$ behavior. While the proton A_1 data do indicate a steep rise at large x, the neutron asymmetry is, within errors, consistent with zero over the measured range [16]. It will be of great interest in future to observe whether, and at which x and Q^2, the A_1 asymmetries start to approach unity.

Another quantity of current interest is the ratio of the neutron to proton F_2 structure functions at large x. There are a number of predictions for this ratio, ranging from 2/3 in the SU(6) symmetric quark model, to 1/4 in broken SU(6) through d quark suppression [17], to 3/7 in broken SU(6) via helicity flip suppression [18]. Although it is well established that the large-x F_2^n/F_2^p data deviate from the SU(6) prediction, they are at present inconclusive about the precise $x \to 1$ limit because of large nuclear corrections in the extraction of F_2^n from deuterium cross sections beyond $x \sim 0.6$.

The ratios of the neutron to proton F_1, F_2 and g_1 structure functions are shown in Fig. 2 as a function of Q^2. While the F_2 ratio varies quite rapidly at low Q^2, beyond $Q^2 \sim 3$ GeV^2 it remains almost Q^2 independent, approaching the asymptotic value $(dG_M^{n2}/dQ^2)/(dG_M^{p2}/dQ^2)$. Because the F_1 n/p ratio depends only on G_M, it remains flat over nearly the entire range of Q^2. At asymptotic Q^2 the model predictions for $F_1(x \to 1)$ coincide with those for F_2; at finite Q^2 the difference between F_1 and F_2 can be used to predict the $x \to 1$ behavior of the longitudinal structure function, or the $R = \sigma_L/\sigma_T$ ratio.

The spin-dependence of the proton vs. neutron duality predictions is also rather interesting. Since A_1^n is zero for all x according to SU(6), the ratio of the neutron to proton g_1 structure functions is also zero in the spin-flavor symmetric limit. The pattern of SU(6) breaking for g_1^n/g_1^p essentially follows that for F_2^n/F_2^p, namely 1/4 in the d quark suppression [17] and 3/7 in the helicity flip suppression [18] scenarios. According to local duality, the g_1 structure function ratio in Fig. 2 approaches the asymptotic limit in Eq.(3c), albeit somewhat slowly, reflecting the relatively slow approach towards unity of the polarization asymmetry in Fig. 1. This may indicate a larger role played by higher twists in g_1 compared with F_2, a result consistent with the analyses of the higher twist corrections to moments of the g_1 [6] and F_2 structure functions [5,19].

It appears to be an interesting coincidence that the helicity retention model [18] prediction of 3/7 is very close to the empirical ratio of the squares of the neutron and proton magnetic form factors, $\mu_n^2/\mu_p^2 \approx 4/9$. Indeed, if one approximates the Q^2 dependence of the proton and neutron form factors by dipoles, and takes $G_E^n \approx 0$, then the structure function ratios are all given by simple analytic expressions, $F_2^n/F_2^p \approx F_1^n/F_1^p \approx g_1^n/g_1^p \to \mu_n^2/\mu_p^2$ as $Q^2 \to \infty$. On the other hand, for the g_2 structure function, which depends on both G_E and G_M at large Q^2, one has a different asymptotic behavior, $g_2^n/g_2^p \to \mu_n^2/(\mu_p(1+\mu_p)) \approx 0.345$.

Of course the reliability of the duality predictions are only as good as the quality of the empirical data on the electromagnetic form factors. While the duality relations are expected to be progressively more accurate with increasing Q^2 [4], the difficulty in measuring form factors at large Q^2 also increases. Experimentally, the proton magnetic form factor G_M^p is relatively well constrained to $Q^2 \sim 30$ GeV^2, and the proton electric G_E^p to $Q^2 \sim 10$ GeV^2. The neutron magnetic form factor G_M^n has been measured to $Q^2 \sim 5$ GeV^2, although the neutron G_E^n is not very well determined at large Q^2 (fortunately, however, this plays only a minor role in the duality relations, with the exception of the neutron to proton g_2 ratio, Eq.(3d)). The fit [15] uses all the available form factor data, together with perturbative QCD constraints on the shapes at $Q^2 \to \infty$. We have tested the sensitivity of the results in Figs. 1 and 2 to different form factor parameterizations [20,21], including adopting simple dipole forms, but find only small effects.

Obviously more data at larger Q^2 would allow more accurate predictions for the $x \to 1$ structure functions, and new experiments at Jefferson Lab [22] and elsewhere will provide valuable constraints. However, the most challenging aspect of testing the validity of the local duality hypothesis is measuring the inclusive structure functions at high enough x. Rapidly falling cross sections as $x \to 1$ mean that only very high luminosity facilities are able to extract these with sufficiently small errors. The most promising possibility at present is the energy-upgraded CEBAF accelerator at Jefferson Lab. Once data on the longitudinal and spin-dependent structure functions at large x become available, a more complete test of local duality between elastic form factors and $x \sim 1$ structure functions can be made.

Along with the spin dependence, unraveling the flavor dependence of duality is also of fundamental importance. Although the local duality relations discussed here are empirical, a more elementary description of the quark–hadron transition requires understanding the transition from coherent to incoherent dynamics and the role of higher twists for individual quark flavors. This is as relevant for all the $N \to N^*$ transition form factors as for the elastic. The flavor dependence can be determined by either studying different hadrons, or tagging mesons in the final state of semi-inclusive scattering in the resonance region [23]. Both the flavor and spin dependence of duality, and more generally the relationship between incoherent (single quark) and coherent (multi-quark) processes, can be addressed with an energy upgrade at Jefferson Lab, which should shed considerable light on the nature of the quark $\to$ hadron transition in QCD.

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FIGURES

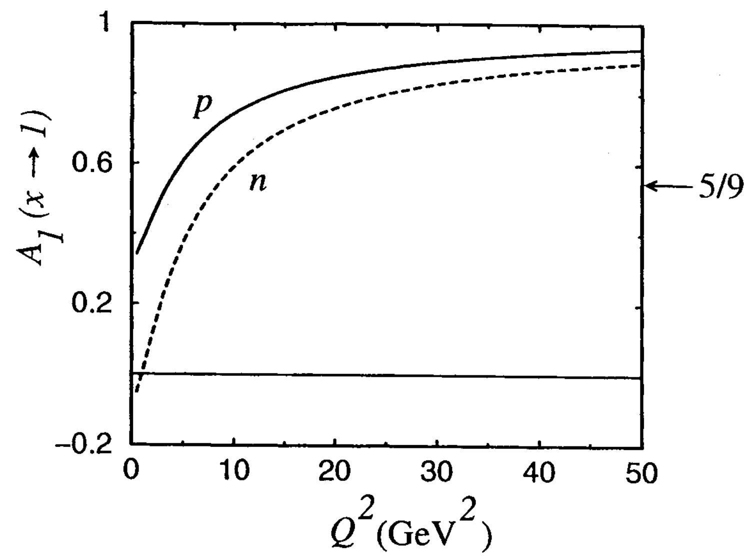

FIG. 1. Polarization asymmetries A_1 for the proton (solid) and neutron (dashed) in the limit $x \to 1$. The SU(6) predictions are 5/9 for p and 0 for n.

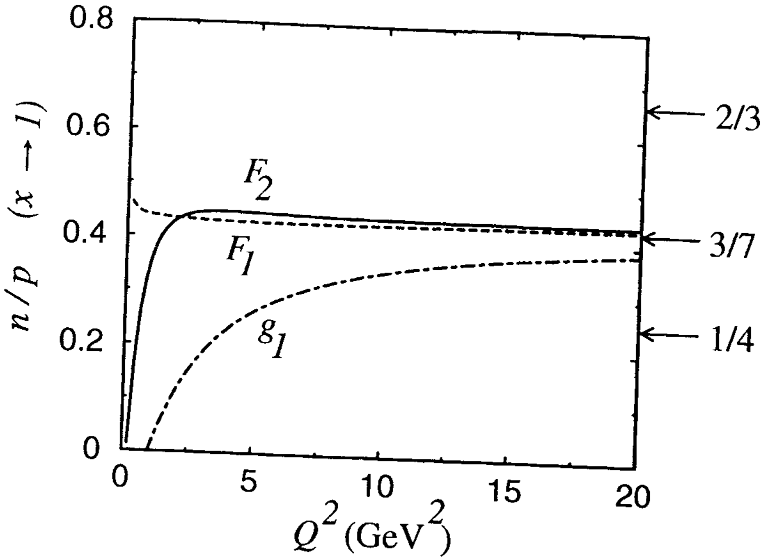

FIG. 2. Neutron to proton ratio for F_1 (dashed), F_2 (solid) and g_1 (dot-dashed) structure functions in the limit $x \to 1$. Several model predictions for F_2 are indicated by the arrows: 2/3 from SU(6), 3/7 from SU(6) breaking via helicity retention, and 1/4 from SU(6) breaking through d quark suppression.